The above equation for ω has no positive roots when $p_{v0} < p_{10}$, nor when $p_{v0} > p_{10}$ provided that the bubble radii satisfy the condition

$$a_0 < \frac{4}{3} a_\sigma, \quad a_\sigma = \sigma/(p_{v0} - p_{10}) \tag{3.9}$$

Therefore the mixture will be stable under these conditions also. The largest radius of the equilibrium bubbles in the case of a superheated liquid $(p_{v0} > p_{10})$ is found from the relation $a_* = 2a_\sigma$.

Thus the vapour-gas-liquid mixture containing bubbles, underheated with respect to the saturation pressure determined at the flat boundary of separation of the phases, is always stable. The superheated mixture is stable if the bubbles are sufficiently small and satisfy the condition (3.9).

Therefore, the simultaneous action of capillary phenomena and phase transitions may lead to violation of the stability of vapour-liquid mixtures containing bubbles, and the pace of the instability in question will basically be limited by the temperature imbalance in the liquid.

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A GENERAL SOLUTION OF THE STATIC PROBLEM OF THE THEORY OF ASYMMETRICAL ELASTICITY*

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A general solution of the homogeneous static relations of the theory of asymmetric elasticity is constructed. The passage to the solution of the classical (symmetric) theory of elasticity is shown, and the form of the general solution for the plane problem is derived.

Certain modifications to the general solution of the equations of equilibrium in the theory of elasticity serve as a basis for formulating various different expressions for the Castigliano functional in the stress functions /1/.

1. When the mass forces and moments are omitted, the static relations of the theory of asymmetric elasticity have the form /2/

$$\nabla \cdot \mathbf{T} = 0, \quad \nabla \cdot \mathbf{M} - \boldsymbol{\epsilon} \cdot\cdot \mathbf{T} = 0 \tag{1.1}$$

where $\mathbf{T}, \mathbf{M}$ are the asymmetric stress and couple stress tensors, respectively, $\boldsymbol{\epsilon}$ is the Levi-Civita tensor and ∇ is the Hamiltonian operator.

Consider the first relation of (1.1). We know /2/ that a tensor whose divergence is equal to zero can be represented in terms of the curl of another tensor. We therefore write ($\mathbf{P}$ is an arbitrary differentiable second-rank tensor)

$$\mathbf{T} = \nabla \times \mathbf{P} \tag{1.2}$$

Relation (1.2) satisfies the first relation of (1.1) identically. Substituting (1.2) into the second relation of (1.1) and taking into account the validity of the transformation

$$\boldsymbol{\epsilon} \cdot\cdot \nabla \times \mathbf{P} = \nabla \cdot \mathbf{I} \cdot\cdot \mathbf{P} - \mathbf{P} \cdot \nabla$$

we can write the second relation of (1.1) in the form

$$\nabla \cdot (\mathbf{M} + \mathbf{P}^T - \mathbf{I} \cdot\cdot \mathbf{P}) = 0 \tag{1.3}$$

where $\mathbf{I}$ is a unit second-rank tensor and T denotes transposition.

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Using the property of the curl of an arbitrary tensor indicated above, we will put the expression within the brackets in (1.3) equal to $\nabla\times\mathbf{Q}$, where $\mathbf{Q}$ is an arbitrary differentiable second-rank tensor. Then

$$\mathbf{M} = \nabla \times \mathbf{Q} - \mathbf{P}^T + \mathbf{II}\cdot\cdot\mathbf{P} \tag{1.4}$$

The above relation satisfies the second equation of (1.1).

Thus we see that the representation of the tensors $\mathbf{T}$ and $\mathbf{M}$ in the form (1.2), (1.4) satisfies the homogeneous equations of statics of the theory of asymmetric elasticity identically.

2. We shall now show that the solution (1.2), (1.4) generalizes the well-known solution of the static equation of the classical theory of elasticity. In the latter theory it is assumed that $\mathbf{M} = 0$. In this case relation (1.4) reduces to the form

$$\mathbf{P} - \mathbf{II}\cdot\cdot\mathbf{P} = \mathbf{Q} \times \nabla \tag{2.1}$$

Let us subtract, from the right-hand side of (2.1), the expression $^3/_2\mathbf{II}\cdot\cdot(\mathbf{Q}-\mathbf{Q})\times\nabla$ identically equal to zero. Then, remembering that

$$\mathbf{II}\cdot\cdot\mathbf{Q}\times\nabla = {}^1/_3\mathbf{II}\cdot\cdot\mathbf{II}\cdot\cdot\mathbf{Q}\times\nabla$$

we can write (2.1) in the form

$$(\mathbf{I} - \mathbf{II}\cdot)\cdot(\mathbf{P} + \mathbf{Q}\times\nabla - {}^1/_2\mathbf{II}\cdot\cdot Q\times\nabla) = 0 \tag{2.2}$$

From (2.2) it follows that

$$\mathbf{P} = -\mathbf{Q}\times\nabla + {}^1/_2\mathbf{II}\cdot\cdot\mathbf{Q}\times\nabla$$

Substituting this expression into (1.2) we obtain

$$\mathbf{T} = -\,\mathbf{Ink}\,\mathbf{Q} + {}^1/_2\nabla\times(\mathbf{II}\cdot\cdot\mathbf{Q}\times\nabla) = -\,\mathbf{Ink}\,(\mathbf{Q} + {}^1/_2\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}\cdot\cdot\mathbf{Q}) = -\,\mathbf{Ink}\,(\mathbf{Q} - \mathbf{Q}^A) = -\,\mathbf{Ink}\,\mathbf{Q}^S \tag{2.3}$$

Here $\mathbf{Ink}$ is the incompatibility operator and the superscripts A and S denote the alternation and symmetrization operations.

Expression (2.3) is a general solution of the static equations of the classical theory of elasticity /2/ and a special case of the solution (1.2) and (1.4).

3. Relations connecting the stress tensor with couple stress tensors by means of two scalar functions, were given in /3/ for the plane problem of the theory of asymmetric elasticity. We shall show that these relations follow directly from (1.2) and (1.4). Let

$$\mathbf{P} = \mathbf{I}_3\mathbf{I}_1\alpha + \mathbf{I}_3\mathbf{I}_2\beta, \quad Q = \mathbf{I}_3\mathbf{I}_3\gamma \tag{3.1}$$

where $\mathbf{I}_1, \mathbf{I}_2$ and $\mathbf{I}_3$ are the Cartesian basis unit vectors, and α, β and γ are scalar functions of the variables x_1, x_2.

Substituting (3.1) into (1.2), (1.4), we obtain

$$\mathbf{T} = \nabla\times(\mathbf{I}_3\mathbf{I}_1\alpha + \mathbf{I}_3\mathbf{I}_2\beta), \quad \mathbf{M} = \nabla\times\mathbf{I}_3\mathbf{I}_3\gamma - \mathbf{I}_1\mathbf{I}_3\alpha - \mathbf{I}_2\mathbf{I}_3\beta \tag{3.2}$$

In expanded form the components of the tensors $\mathbf{T}$ and $\mathbf{M}$ will be written according to (3.2), in the form

$$\mathbf{T} = \begin{Vmatrix} \alpha_{,2} & \beta_{,2} & 0 \\ -\alpha_{,1} & -\beta_{,1} & 0 \\ 0 & 0 & 0 \end{Vmatrix}, \quad \mathbf{M} = \begin{Vmatrix} 0 & 0 & \gamma_{,2} - \alpha \\ 0 & 0 & -\gamma_{,1} - \beta \\ 0 & 0 & 0 \end{Vmatrix} \tag{3.3}$$

where the subscript following the coma denotes differentiation with respect to the corresponding coordinate. We see that the expressions obtained contain all independent components of the stress state of the plane problem of the theory of asymmetric elasticity. They satisfy its static relations identically and by that token they represent their general solution, which is expressed in terms of three scalar function α, β and γ.

Let us further impose on (3.2) an additional condition of compatibility of deformations of the form

$$\nabla\times\mathbf{M} = 0 \tag{3.4}$$

Relation (3.4) is satisfied identically if the tensor $\mathbf{M}$ for the plane problem can be written in the form

$$\mathbf{M} = \nabla\mathbf{I}_3\varphi, \quad \varphi \equiv \varphi(x_1, x_2) \tag{3.5}$$

Equating (3.2) with (3.5) we find that

$$\mathbf{I}_3\mathbf{I}_1\alpha + \mathbf{I}_3\mathbf{I}_2\beta = (\gamma\mathbf{I}_3\mathbf{I}_3\times\mathbf{I} - \varphi\mathbf{I}_3\mathbf{I})\cdot\nabla \tag{3.6}$$

Substituting (3.6) into the first relation of (3.2) we obtain

$$\mathbf{T} = \nabla\times(\gamma\mathbf{I}_3\mathbf{I}_3\times\mathbf{I} - \varphi\mathbf{I}_3\mathbf{I})\cdot\nabla \tag{3.7}$$

Expressions (3.5) and (3.7) connect the tensors $\mathbf{T}$ and $\mathbf{M}$ using two scalar stress functions γ and φ. The relations satisfy identically the equations of equilibrium (1.1) and one of the conditions of compatibility of the deformations written in the form (3.4). Relations (3.5) and (3.7) can be written in expanded form as

$$\mathbf{T} = -\begin{Vmatrix} -(\gamma_{,22}+\varphi_{,12}) & (\gamma_{,12}+\varphi_{,22}) & 0 \\ (\gamma_{,12}-\varphi_{,11}) & -(\gamma_{,11}+\varphi_{,12}) & 0 \\ 0 & 0 & 0 \end{Vmatrix}, \quad \mathbf{M} = \begin{Vmatrix} 0 & 0 & \varphi_{,1} \\ 0 & 0 & \varphi_{,2} \\ 0 & 0 & 0 \end{Vmatrix}$$

which agrees completely with the solution given in /3/.

We note that the general solutions of the equations of equilibrium of the theory of asymmetric elasticity obtained here, enable us to solve specific problems.

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A HOLE IN A PLATE, OPTIMAL FOR ITS BIAXIAL EXTENSION - COMPRESSION*

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An outline of a hole of equal strength in an elastic plate loaded at infinity by mutually perpendicular tensile and compressive forces, is obtained. It is shown that under these conditions the hole is bounded by a contour with corners, and its form is found by numerical methods. It turns out that the contour is very close to rectangular, with slightly rounded sides, whose ratio depends on the load.

Let a thin unbounded plate made of a homogeneous, isotropic, linearly elastic material, occupy the region S in the plane of the complex variable $z = x + iy$, and let it be weakened by a hole with an arbitrary, piecewise-smooth boundary Γ enclosing the origin of the Cartesian coordinate system XOY. Specified forces P_x and P_y, $P_y/P_x = \lambda$ act along the axes of this system, and the hole is load-free.

The stress state of the plate is found from the solution of the homogeneous boundary value problem /1/

$$\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)} = 0; \quad t \in \Gamma \tag{1}$$

where t is the complex coordinate of any point on the contour, and the Muskhelishvili potentials $\varphi(z)$ and $\psi(z)$ holomorphic in $S+\Gamma$ have the following asymptotic form at infinity:

$$4\varphi(z) = P_x(1+\lambda)z + O(|z|^{-1}) \tag{2}$$

$$2\psi(z) = P_x(\lambda - 1)z + O(|z|^{-1})$$

We can also consider problem (1) with the right-hand side $f(t) = -{}^1/_2 P_x(1+\lambda)t - {}^1/_2 P_x(\lambda-1)\bar{t}$, with respect to potentials decreasing at infinity.

The equal-strength boundary of the hole is determined, as we know /2/, by the condition

$$\sigma_\tau(t) = \mathrm{const}, \quad t \in \Gamma \tag{3}$$

expressing the constancy of the tangential component of the stress tensor on it (the normal component of this tensor is, according to the boundary condition, equal to zero).

Such a boundary represents the solution of the problem of optimal design of a hole in a plate relative to either of the two optimizing functionals.

A. The potential energy of the plate deformation functional. First the integral functional is regularized by subtracting from its density a constant term corresponding to the homogeneous stress field of a solid plate. The functional expresses the weakening effect of a cutout. In /3/ it is shown that when (3) holds, a stationary point (for small variations in the form of Γ) of the functional is reached, provided that the area of the hole is given.

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